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Cosmology with Time-Varying G

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It is shown that a Universe with a time-varying gravitational "constant" G necessarily implies creation if the rest mass of matter particles m_p is constant. In this case, from Einstein's field equations, the conditions for energy-momentum propagation are $\nabla \cdot (GT^{\mu\nu}) = 0$ from which matter and photon propagation equations are derived. Free matter particle propagation is not affected by creation that is given by $GN_p m_p = \text{const}$, where N_p is the number of matter particles within a proper volume. This relation introduces explicitly the rest mass of the Universe into the field equations. Free photon propagation is affected by creation that is given by $GN_\gamma \nu R = \text{const}$, where N_γ is the number of photons within a proper volume, which is the cosmic red shift law. Conservation of the cosmic background photon distribution determines photon creation as $G^3 N_\gamma^4 = \text{const}$. The results are applied to the case $G \div t^{-1}$ equivalent to $N_p \div t$.

It is found that at an age $t = 10^{-40} t_0$, of the order light takes to travel a proton size, Planck's units become of the order of the proton's mass m_p, size r_p, and time r_p/c. Hence, matter particles at this age are quantum black holes. Evaporation of these quantum black holes at this age gives a background blackbody radiation that, red shifted to present time t_0, gives the present cosmic microwave background.

A cosmological model of the Friedmann type is constructed. The red shift versus distance relation is derived taking into account creation. Using a Hubble's constant $H_{\text{obs}} = 50\ \text{km sec}^{-1}\ \text{Mpc}^{-1}$ and a deceleration parameter $q_{\text{obs}} = 1.0$ the model is of the type $k = 1$ and gives a present age $t_0 = 6.81 \times 10^9$ yr, consistent with Uranium model ages. Thus, the three results for the age of the Universe, i.e., radioactive decay, Hubble's constant, and stellar evolution are brought together in this creation model. The matter-dominated era occurs for $t > 7.6 \times 10^{-3} t_0$, while the radiation-dominated era occurs for $7.6 \times 10^{-3} t_0 > t > 10^{-40} t_0$. The origin of the Universe is placed at this last limit, which is Planck's time at the corresponding G, consisting of quantum black holes at a temperature $T_i \simeq 3 \times 10^{11}$ K.

1. INTRODUCTION

Much work has been done in the past using the hypothesis that the gravitational "constant" G varies with time (e.g., Dirac, 1937); Brans and

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Dicke, 1961), and a current assumption is that G varies as t^{-1}. This assumption is suggested by the numerical coincidence of two large numbers: the ratio of electric and gravitational forces between an electron and a proton, which is of the order of 10^{40}, and the ratio of the age of the Universe to the time light takes to travel the size of an elementary particle. Dirac generalized this result to say that any number which is a power of 10^{40} will be time dependent to the same power, which constitutes the large number hypothesis, and for the number of particles in the Universe, $N_p \simeq 10^{80}$, implies $N_p \div t^2$. This last relation has been used by some authors, e.g., Adams (1982). In the work presented here we take the view that G varies as t^{-1} and we show that this is enough to determine all other cosmological parameters, N_p, ρ, R, etc., in such a way that the large number hypothesis is not substantiated except for the assumed law $G \div t^{-1}$. Also we take the view that Nature has only one set of units, the same for microphysics as for macrophysics, and classical general relativity is used up to Planck's distance and time.

From Einstein's field equations we derive the conditions of energy-momentum propagation as $\nabla \cdot (GT^{\mu\nu}) = 0$, retaining G as a function of time only, instead of the usual $\nabla \cdot T^{\mu\nu} = 0$. The energy equation gives the condition $GN_pm_p = \text{const}$ for matter particles in the Universe with zero pressure. For a constant rate of creation, $N_p \div t$, one has $G \div t^{-1}$, which is one of Dirac's hypotheses. In this way a large cosmic number, the rest mass N_pm_p of matter in the Universe, is explicitly introduced into the Einstein field equations, through G.

Next we find the propagation equations for matter particles and photons following a technique already used by Adams (1983), though we use $\nabla \cdot (GT^{\mu\nu}) = 0$ and assume one and only one natural set of units for all physics. We show that matter particle propagation is not affected by creation and that the result $GN_pm_p = \text{const}$ is a general one. On the contrary, photon propagation is affected by creation and the corresponding red shift law is derived, which differs from the classical one if G varies with time.

Conservation of the photon distribution function in the cosmic background, a condition also used by Adams (1983), determines the creation rate of photons as $G^3N_\gamma^4 = \text{const}$, where N_γ is the number of photons in the Universe. We see that a time-varying G also implies photon creation, as well as matter particle creation.

Next follows the analysis of the role that Planck's units have when we consider G varying with time. We show that they become of the order of the proton's dimensions at an age of the Universe such that light has had time to travel only the size of the proton, which is also Planck's time then. Hence, at this age the protons (or neutrons) were quantum black holes. The present 2.7-K background radiation blue-shifted to this age results in a blackbody radiation similar in temperature to the one the quantum black holes tend to give by "evaporation." Radiation and matter particles become isolated quantum black holes at this age where fluctuations in space–time are the dominant feature (Wheeler, 1970, 1974).

Finally we present a model of the Universe of the Friedmann type that fits the previous relations. In the case of free matter particles and free photons, two universal constants are found and used to integrate the field equation for the cosmic scale factor $R(t)$. They are the result of the propagation law for particles, $GN_pm_p = \text{const}$, and the cosmic red shift law with creation, $GN_\gamma \nu R = \text{const}$. In terms of matter and radiation densities these universal constants are $G\cdot\rho_p\cdot R^3$ and $G\cdot\rho_\gamma\cdot R^4$. The distance red shift relation is derived taking into account creation, and with present values for the Hubble's constant and the deceleration parameter the model is completely determined giving a value for the present age of the Universe of $t_0 = 6.81\times 10^9$ yr. This is also the uranium age for prompt production.

When Planck's time is of the order of the age of the Universe, and, we find, also of the order of the time light takes to travel the proton size, one has a picture that we interpret as the initial condition of the Universe.

2. TIME VARYING G VERSUS CREATION

We will show that a Universe with a time varying G necessarily implies creation given by $GN_pm_p = \text{const}$.

From Einstein's field equations (e.g., Weinberg, 1972),

$$R^{\mu\nu} - \tfrac{1}{2}g^{\mu\nu}R = -8\pi G T^{\mu\nu} \tag{1}$$

the application of the Bianchi identities give the covariant divergence of the left-hand side of (1) equal zero. Then, one obtains the conditions of energy-momentum conservation which give the equations of motion, with $G = \text{const}$, as

$$T^{\mu\nu}{}_{;\nu} = 0 \tag{2}$$

With a time-varying G one has the conditions of energy–momentum propagation as

$$(GT^{\mu\nu});\, \nu = 0 \tag{3}$$

For a perfect fluid one has

$$T^{\mu\nu} = pg^{\mu\nu} + (p+\rho)U^\mu U^\nu \tag{4}$$

which is the form of the energy-momentum tensor for the Universe, where p and ρ are the pressure and energy density functions of time t only. Following the standard way to find the energy-conservation equation, one has for a comoving observer the velocity four-vector U^μ defined as

$$U^t = 1$$
$$U^i = 0 \tag{5}$$

where t means time component and i space component. Substitution of (4) and (5) into (3) gives three equations trivially satisfied for $\mu = i$, while for $\mu = t$ we get one of the fundamental equations of dynamical cosmology, the energy-conservation equation that now reads

$$R^3 \frac{d}{dt}(Gp) = \frac{d}{dt}[G(\rho + p)R^3] \tag{6}$$

where we have used a Robertson-Walker metric for a homogeneous and isotropic Universe with scale factor $R(t)$. Equation (6) gives the equivalent

$$Gpd(R^3) + d(G\rho R^3) = 0 \tag{7}$$

A common approximation is to make use of the observation that the present energy density of the Universe is dominated by nonrelativistic matter with negligible pressure, so that with $p \ll \rho$ one has from (7)

$$G\rho R^3 \simeq \text{const} \tag{8}$$

or, equivalently,

$$GN_p m_p \simeq \text{const} \tag{9}$$

In the next section we show that (9) must be exactly a constant and valid also for relativistic matter particles as long as they are free. The result implies that if G varies with time N_p also varies with time, m_p kept constant, and therefore there is creation. It also implies that G is determined by the presence of matter particles in the Universe. One can say from (9) that the value of G is inversely proportional to the rest mass (energy) of the Universe.

Throughout this work we assume a constant creation rate, i.e., $N_p \div t$. Apart from simplicity, it is an assumption in line with one of Dirac's (1937) large numbers argument: taking the ratio of the electric force to the gravitational force between an electron and a proton one has

$$\frac{e^2}{Gm_e m_p} = 0.23 \times 10^{40} \tag{10}$$

while the ratio of the age of the Universe t_0 to the time light takes to travel a classical electron radius is, with $t_0 = 6.81 \times 10^9$ yr,

$$\frac{t_0}{e^2/m_e c^3} = 2.3 \times 10^{40} \tag{11}$$

The assumptions that the two numbers (10) and (11) are nearly equal due to a linear cosmological time dependence, with e, m_e, m_p, and c constants, implies $G \div t^{-1}$ and from (9) $N_p \div t$ i.e., a constant creation rate. Since N_p is of order 10^{80}, Dirac also made the assumption that $N_p \div t^2$, an assumption that we can not make in view of the energy equation (9).

3. PROPAGATION EQUATIONS

3.1. Matter Propagation Equation

The particle propagation equation can be obtained using a field-to-particle technique in a similar way as done by Adams (1983), except that we take the energy tensor for any closed system to satisfy $(GT^{\mu\alpha})_{;\alpha}=0$ instead of $T^{\mu\alpha}{}_{;\alpha}=0$. Also we use standard coordinate covariant derivatives (semicolon derivative) and take the view that there is only one natural unit standard, valid for microscopic and for macroscopic quantities.

Taking a curve C lying inside a space-time region with $T^{\mu\alpha}\neq 0$ and constructing Gaussian coordinates along C, the line element is

$$d\tau^2 = dt^2 - h_{ij}\, dx^i\, dx^j \tag{12}$$

The four-dimensional volume element is

$$d_4V = (1/4!)(-g)^{1/2}\mathscr{E}_{\mu\alpha\sigma\lambda}\, dx^\mu\, dx^\alpha\, dx^\sigma\, dx^\lambda \tag{13}$$

where $\mathscr{E}_{\mu\alpha\sigma\lambda}$ is the alternating symbol and $\mathscr{E}_{0123}=1$. On C one has

$$d_4V = (1/3!)(-g)^{1/2}\mathscr{E}_{ijk}\, dx^i\, dx^j\, dx^k\, dt = dV\, dt \tag{14}$$

with $\mathscr{E}_{123}=1$ and dV the three-dimensional proper volume element for any $t=\text{const}$ hypersurface on C. Defining C parametrically $x^\mu=\zeta^\mu(k)$, k being a strictly monotone parameter, one has the tangent vector to C

$$r^\mu = \frac{d\zeta^\mu}{dk} \tag{15}$$

Surrounding C by an arbitrarily small world tube with three-dimensional volume V at time t, and such that $T^{\mu\alpha}$ is zero on the boundary, we assume that at any time t one can choose V small enough to have

$$r^0 \int_V \omega(x) GT^{\mu\alpha}(x)\, dV = \omega(t,\zeta^i) Gt^{\mu\alpha}(t) \tag{16}$$

for any arbitrary function $\omega(x)$ having a Taylor's series expansion about C. It can be seen that $t^{\mu\alpha}$ has the same tensorial properties as $T^{\mu\alpha}$ on C.

The conditions of energy–momentum propagation (3) are

$$(GT^{\mu\alpha})_{;\alpha} = \frac{1}{h^{1/2}}(h^{1/2}GT^{\mu\alpha})_{,\alpha} + GT^{\lambda\alpha}\Gamma^{\mu}_{\lambda\alpha} = 0 \tag{17}$$

where $h = (-g)^{1/2}$. Multiplying the above expression by the arbitrary function $\omega(x)$ one has

$$\frac{1}{h^{1/2}}(h^{1/2}\omega GT^{\mu\alpha})_{,\alpha} - \omega_{,\alpha}GT^{\mu\alpha} + \omega GT^{\lambda\alpha}\Gamma^{\mu}_{\lambda\alpha} = 0 \tag{18}$$

We can apply (16) to the above result and obtain

$$r^0 \int_v \frac{1}{h^{1/2}}(h^{1/2}\omega GT^{\mu\alpha})_{,\alpha}\, dV - \omega_{,\alpha}Gt^{\mu\alpha} + \omega Gt^{\lambda\alpha}\Gamma^{\mu}_{\lambda\alpha} = 0 \tag{19}$$

Let us develop the integral term in (19). Using (14) one has

$$r^0 \int_v \frac{1}{h^{1/2}}(h^{1/2}\omega GT^{\mu\alpha})_{,\alpha}\, dV = r^0 \frac{d}{dt}\int_v \omega GT^{\mu 0}\, dV + \frac{r^0}{3!}\int_v \frac{\partial}{\partial x^n}(h^{1/2}\omega GT^{\mu n})\mathscr{E}_{ijk}\, dx^i \cdot dx^j \cdot dx^k$$

where the last integral above vanishes since $T^{\mu\alpha}$ is zero on the boundary of V. On the other hand one has using (15) and (16)

$$r^0 \frac{d}{dt}\int_v GT^{\mu 0}\, dV = \frac{d}{dk}\left(\frac{\omega Gt^{\mu 0}}{r^0}\right) = \frac{Gt^{\mu 0}}{r^0}\omega_{,\alpha}r^\alpha + \omega\frac{d}{dk}\left(\frac{Gt^{\mu 0}}{r^0}\right)$$

Hence, substitution of this expression for the integral in (19) gives

$$\left(\frac{Gt^{\mu 0}}{r^0}r^\alpha - Gt^{\mu\alpha}\right)\omega_{,\alpha} + \left[\frac{d}{dk}\left(\frac{Gt^{\mu 0}}{r^0}\right) + Gt^{\lambda\alpha}\Gamma^{\mu}_{\lambda\alpha}\right]\omega = 0 \tag{20}$$

Since ω is arbitrary, the factors of $\omega_{,\alpha}$ and ω in (20) must be zero. Then

$$\frac{t^{\mu 0}}{r^0}r^\alpha = t^{\mu\alpha} \tag{21}$$

$$\frac{d}{dk}\left(\frac{Gt^{\mu 0}}{r^0}\right) + Gt^{\lambda\alpha}\Gamma^{\mu}_{\lambda\alpha} = 0 \tag{22}$$

Taking into account that r^α and $t^{\mu\alpha}$ are tensors on C, one can define t^μ as follows:

$$t^\mu = \frac{t^{\mu 0}}{r^0} \tag{23}$$

and substitution in (21) and (22) gives

$$t^{\mu} r^{\alpha} = t^{\mu\alpha} \tag{24}$$

$$\frac{d}{dk}(Gt^{\mu}) + Gt^{\lambda} r^{\alpha} \Gamma^{\mu}_{\lambda\alpha} = 0 \tag{25}$$

Using (15) in (25) one gets

$$r^{\alpha}(Gt^{\mu})_{,\alpha} + Gt^{\lambda} r^{\alpha} \Gamma^{\mu}_{\lambda\alpha} = 0 \tag{26}$$

i.e.,

$$(Gt^{\mu})_{;\alpha} r^{\alpha} = 0 \tag{27}$$

Equations (24) and (27) are the propagation equations that have been determined from (3). For matter particles $T^{\lambda}_{\lambda} \neq 0$ and hence $t^{\lambda}_{\lambda} \neq 0$. Then, from (24) one has

$$t^{\lambda} r_{\lambda} \neq 0 \tag{28}$$

On the other hand $T^{\mu\alpha} = T^{\alpha\mu}$ and hence $t^{\mu\alpha} = t^{\alpha\mu}$. Then

$$t^{\mu} r^{\alpha} = t^{\alpha} r^{\mu} \tag{29}$$

$$t^{\mu} r^{\lambda} r_{\lambda} = t^{\lambda} r_{\lambda} r^{\mu} \tag{30}$$

Since on $C t^{\mu} \neq 0$ and C has a tangent always, i.e., $r^{\mu} \neq 0$, (28) and (30) imply

$$r^{\lambda} r_{\lambda} \neq 0 \tag{31}$$

i.e., C is always timelike since we are treating matter particles. Taking into account (28) and (31) we can define a new strictly monotone parameter σ on C

$$\frac{d\sigma}{dk} = \frac{r^{\lambda} r_{\lambda}}{r^{\alpha} t_{\alpha}} \tag{32}$$

Hence, from (30) and (15)

$$t^{\mu} = r^{\mu} \frac{dk}{d\sigma} = \frac{d\zeta^{\mu}}{d\sigma} \tag{33}$$

and the propagation equation (27) becomes

$$(Gt^{\mu})_{;\alpha} t^{\alpha} = 0 \tag{34}$$

Let us find the meaning of t^{μ}. Taking into account (16) and (23) one has

$$t^{0} = \frac{t^{00}}{r^{0}} = \int_{v} T^{00}\, dV = NP_{0} \tag{35}$$

where P_0 is the energy per particle and N the number of particles inside the proper volume V. Since $T^{\mu\alpha}$ is not zero only inside a very thin world tube, the N particles inside V have almost the same four-momentum P^μ. Therefore, from (33) and (35) we can define a new path parameter λ such that

$$P^\mu = \frac{dx^\mu}{d\lambda} = \frac{d\sigma}{d\lambda} t^\mu = \frac{t^\mu}{N} \tag{36}$$

Substitution of (36) into (34) finally gives

$$(GNP^\mu)_{;\alpha} P^\alpha = 0 \tag{37}$$

The above equation is the free particle propagation law and coincides with the one obtained by Adams (1983) if $G = \text{const}$. Using $P^\mu = m_p U^\mu$ in (37), with the rest mass m_p constant, gives

$$U^\mu{}_{;\alpha} U^\alpha + \frac{d \ln(GN)}{dt} U^\mu U^0 = 0 \tag{38}$$

and contraction with U_μ then gives

$$\frac{d \ln(GN)}{dt} = 0, \qquad \text{i.e., } GN = \text{const} \tag{39}$$

Hence, from (38) and (39) one obtains as the equations of motion of free particles the geodesic equation

$$U^\mu{}_{;\alpha} U^\alpha = 0 \tag{40}$$

We have shown in (9) for a Universe of nonrelativistic matter particles one has $GN_p = \text{const}$. Now (39) is a generalization of this relation and the free particle propagation law (37) can be written

$$P^\mu{}_{;\alpha} P^\alpha = 0 \tag{41}$$

The above result proves that free matter particle propagation is not affected by creation.

It is well known that the classical nonrelativistic continuity and Euler's equations for a perfect neutral fluid can be obtained from (2) and (4). Using (3) instead of (2) gives

$$\frac{\partial(G\rho)}{\partial t} + \nabla \cdot (G\rho\mathbf{v}) = 0 \tag{42}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) v = -\frac{\nabla Gp}{G\rho} = -\frac{\nabla p}{\rho} \tag{43}$$

It is seen that Euler's equation (43) remains the same whether there is creation or not. This means that for nonrelativistic classical neutral fluids the momentum equation is not affected by creation. On the other hand, the continuity equation (42) can be written equivalently,

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\mathbf{v})=-\rho\frac{d\ln G}{dt}=\frac{\rho}{t} \tag{44}$$

which is the classical nonrelativistic equation with a source term on the right-hand side due to G varying with time.

For a static fluid, $\mathbf{v}=0$ in (42) gives $G\rho=\text{const}$ and in a unit volume one has

$$GN=\text{const} \tag{45}$$

Hence, classically a time-varying G also implies creation with the same law as predicted by general relativity.

3.2. Photon Propagation Equation

We use the field-to-particle technique for photons, as done by Adams (1983), except that here the symmetric energy tensor satisfies $(GT^{\mu\alpha})_{;\alpha}=0$. The propagation equations (24) and (27) are also valid for photons, i.e.,

$$t^{\mu}r^{\alpha}=t^{\mu\alpha} \tag{24}$$

$$(Gt^{\mu})_{;\alpha}r^{\alpha}=0 \tag{27}$$

Now, for photons $T^{\lambda}_{\lambda}=0$ and then $t^{\lambda}_{\lambda}=0$; hence from (24)

$$t^{\lambda}r_{\lambda}=0 \tag{46}$$

and (30) gives now

$$r^{\lambda}r_{\lambda}=0 \tag{47}$$

i.e., C is a null curve as must be the case for photons. If we contract (29) with t_{μ} and use (46) we see that t^{μ} is null. Then, from (46) and (47) t^{μ} and r^{μ} must be proportional for some scalar function $A(x)\neq 0$,

$$t^{\mu}=Ar^{\mu} \tag{48}$$

We can define a new strictly monotone parameter σ on C such that

$$\frac{d\sigma}{dk}=\frac{1}{A(x)} \tag{49}$$

then from (48) and (15)

$$t^{\mu}=\frac{dk}{d\sigma}r^{\mu}=\frac{d\zeta^{\mu}}{d\sigma} \tag{50}$$

Hence, the propagation equation (27) becomes

$$(Gt^{\mu})_{;\alpha}t^{\alpha}=0 \tag{51}$$

which is the same as for matter particles (34). For the meaning of t^{μ} we can follow the same steps (35) and (36), and substitution in (51) gives

$$(GN_{\gamma}P^{\mu})_{;\alpha}P^{\alpha}=0 \tag{52}$$

i.e., the same equation as for matter particles (37) except that now we have N_{γ} as the number of photons inside the proper volume V and P^0 is the energy per photon. From (52) we see that the photon propagation law is affected by photon creation, $N_{\gamma}(t)$, unless $GN_{\gamma}=\text{const}$, which is not the case as shown later on the basis of the constancy of the 2.7-K photon distribution function throughout the history of the Universe.

4. THE RED SHIFT LAW

The cosmological red shift law can be derived using the photon propagation equation (52). Taking the cosmological metric

$$d\tau^2=dt^2-R^2(t)h_{ij}\,dx^i\,dx^j \tag{53}$$

$$0=\Gamma^i_{00}=\Gamma^0_{00}=\Gamma^0_{0i}, \qquad \Gamma^i_{0j}=\delta^i_j\dot{R}/R \tag{54}$$

with $\dot{R}$ meaning the time derivative of $R(t)$, one has from (52) and (36)

$$\begin{aligned}0&=(GN_{\gamma}P_0)_{;\alpha}P^{\alpha}=(GN_{\gamma}P_0)_{,\alpha}P^{\alpha}-\Gamma^{\lambda}_{0\alpha}GN_{\gamma}P_{\lambda}P^{\alpha}\\&=\frac{d(GN_{\gamma}P_0)}{d\lambda}+\frac{\dot{R}}{R}GN_{\gamma}P_0P^0=\frac{d(GN_{\gamma}P_0)}{d\lambda}+GN_{\gamma}P_0\frac{d\ln R}{d\lambda}\end{aligned} \tag{55}$$

Hence one obtains

$$GN_{\gamma}P_0R=\text{const} \tag{56}$$

Relative to a comoving observer $P_0=h\nu$ and (56) gives

$$GN_{\gamma}\nu R=\text{const} \tag{57}$$

as the cosmic red shift law. It differs from the classical one $\nu R=\text{const}$ by the factor GN_{γ}, which next we show is not constant.

5. CONSERVATION OF THE BACKGROUND PHOTON DISTRIBUTION FUNCTION

The number density of photons in phase space is the distribution function $f(\bar{x},\bar{p})$ defined as

$$f=\frac{N_{\gamma}}{VP} \tag{58}$$

where N_γ is the number of particles occupying the volume V and having local Lorentz momentum components $\bar{p}$ in the range $\bar{p}_0 \pm \Delta\bar{p}/2$. We want to determine

$$\frac{df}{d\lambda}=f\frac{d\ln N_\gamma}{d\lambda}-f\frac{d\ln VP}{d\lambda} \tag{59}$$

along the path of an ensemble of free photons. Again, we follow Adams' treatment (Adams, 1983), but with the photon propagation law (52) derived here, giving

$$\frac{dp^i}{d\lambda}=-p^i\frac{d\ln(GN_\gamma)}{d\lambda} \tag{60}$$

where the semicolon derivative in (52) has been substituted by a colon derivative since we are in a local Lorentz frame with λ satisfying (36). Consider a thin beam of monochromatic photons that occupy a small rectangular phase space region at λ. A photon that at λ was at $(\bar{x}, \bar{p})$, at $\lambda + d\lambda$ will be at

$$(\bar{x}+\bar{p}\,d\lambda, \bar{p}+A\bar{p}\,d\lambda) \tag{61}$$

where from (60) we have

$$A=-\frac{d\ln(GN_\gamma)}{d\lambda} \tag{62}$$

The phase space volume at λ is

$$VP_{\text{at }\lambda}=\Delta x\Delta y\Delta z\Delta p^x\Delta p^y\Delta p^z \tag{63}$$

while at $\lambda + d\lambda$, from (61) is

$$VP_{\text{at }\lambda+d\lambda}=\Delta x\Delta y\Delta z\Delta p^x\Delta p^y\Delta p^z(1+3A d\lambda) \tag{64}$$

Hence one has

$$\frac{d\ln VP}{d\lambda}=3A=-\frac{d\ln(GN_\gamma)^3}{d\lambda} \tag{65}$$

and substituting in (59) one obtains

$$\frac{df}{d\lambda}=f\frac{d\ln N_\gamma}{d\lambda}+f\frac{d\ln(GN_\gamma)^3}{d\lambda}=f\frac{d\ln(G^3N_\gamma^4)}{d\lambda} \tag{66}$$

Now, if the 2.7-K cosmic photon distribution f is cosmological in origin and has been propagating without collisions for most of the age of the Universe, then (66) must be zero to conserve f. In particular if it is blackbody today this means that it has been always blackbody. For this to happen one has from (66)

$$G^3N_\gamma^4=\text{const} \tag{67}$$

We note that the argument is independent of the type of the distribution function. If the cosmic photon background is cosmological in origin and collisions have been unimportant, then (67) must hold and it determines N_γ which can be interpreted as the number of photons occupying a volume V.

The cosmic red shift law (57) with (67) is given by

$$\frac{\nu R}{N_\gamma^{1/3}} = \text{const} \tag{68}$$

or in terms of G

$$G^{1/4}\nu R = \text{const} \tag{69}$$

Assuming $G \div t^{-1}$ one has

$$\frac{\nu}{\nu_0} = \frac{R_0}{R}\left(\frac{t}{t_0}\right)^{1/4} \tag{70}$$

From (52) and (67) we see that photon propagation is affected by creation, and from (70) we see that creation blue-shifts the free photons. In the flat space-time limit, $R = \text{constant}$ in (70) gives

$$\frac{\nu}{\nu_0} = \left(\frac{t}{t_0}\right)^{1/4} \tag{71}$$

which means that the color of a laser beam gets bluer as it propagates, if creation of photons is present, a condition found by Adams (1983) with a different law.

6. THE ROLE OF PLANCK'S MASS, LENGTH, AND TIME

If we suppose that there exists a particle of mass m^* such that it has a gravitational length equal to its Compton length, we get Planck's units, i.e.,

$$\begin{aligned} m^* &= \left(\frac{\hbar c}{G}\right)^{1/2} = 2.18 \times 10^{-5}\ \text{g} \\ l^* &= \left(\frac{\hbar G}{c^3}\right)^{1/2} = 1.62 \times 10^{-33}\ \text{cm} \\ t^* &= \left(\frac{\hbar G}{c^5}\right)^{1/2} = 5.39 \times 10^{-44}\ \text{sec} \end{aligned} \tag{72}$$

At the Planck scale of distances l^*, Wheeler (1970, 1974), describes quantum general relativity predicting violent fluctuations in the geometry, quantum fluctuations describing gravitational collapse all the time and everywhere. A particle having Planck's units is in fact a black hole of quantum mechanical size. If we compare Planck's units with the proton's dimensions,

$$\begin{aligned} m_p &= 1.07\times 10^{-24}\ \mathrm{g} \\ r_p &= \frac{e^2}{m_e c^2} = 2.82\times 10^{-13}\ \mathrm{cm} \\ r_p/c &= 9.40\times 10^{-24}\ \mathrm{sec} \end{aligned} \tag{73}$$

then m^* is very large and l^*, t^* very small. The possibility that there exists a fundamental length of order l^* has been used by many authors, e.g., Ginzburg (1976), in relation to many problems, in particular, singularities in the general relativity theory and cosmology. If we take into account a time-varying $G \div t^{-1}$ we get from (72)

$$\begin{aligned} m^* &= 2.18\times 10^{-5}\left(\frac{t}{t_0}\right)^{1/2}\ \mathrm{g} \\ l^* &= 1.62\times 10^{-33}\left(\frac{t_0}{t}\right)^{1/2}\ \mathrm{cm} \\ t^* &= 5.39\times 10^{-44}\left(\frac{t_0}{t}\right)^{1/2}\ \mathrm{sec} \end{aligned} \tag{74}$$

Now, from (10) and (11) at time $t=10^{-40}t_0$ the electric forces become of the order of the gravitational ones and light just had time to travel only about the size of the proton. On the other hand, at $t=10^{-40}t_0$ Planck's units (74) become of the order of the protons dimensions (73). This age is

$$t_i = 10^{-40}t_0 = 2.15\times 10^{-23}\ \mathrm{sec} \tag{75}$$

with $t_0 = 6.81\times 10^9$ yr, i.e., t_i is of the order of r_p/c. This means that at this time matter particles were quantum blackholes which tend to evaporate giving blackbody radiation of typical wavelength $\lambda_{bb} = 2\pi r_p$, i.e.,

$$\lambda_{bb} = 2\pi r_p = 1.8\times 10^{-12}\ \mathrm{cm} \tag{76}$$

The time for evaporation t_e can be obtained in order of magnitude equating total emission to rest energy. Since the negative self-gravitational potential energy of a proton at t_i is of the order of its rest energy, one has

$$\rho_{bb}\frac{c}{4}4\pi r_p^2 t_e \lessapprox m_p c^2 \tag{77}$$

and since

$$\rho_{bb} = \frac{1.37\times 10^{-16}}{\lambda_{bb}^4} \tag{78}$$

one obtains from (76), (77), and (78) $t_e \leqslant 1.5 \times 10^{-20}$ sec. Hence, the matter particles tend to evaporate up to an age of the Universe less than 1.5×10^{-20} sec. As soon as G decreases with time we have

$$\frac{Gm_p}{c^2} < r_p \tag{79}$$

and the matter particles are no longer blackholes.

In the next sections we take a model of the Universe to get $R(t)$, the scale factor for expansion, and applying the redshift law (70) we show that the initial blackbody radiation from the quantum blackholes approximately gives the present 2.7-K radiation background.

Thus, the role of Planck's units is important in time-varying G theories. In the case $G \div t^{-1}$ if we consider the present radiation background and the matter particles in the Universe, and go back in time up to $t = r_p/c$, then matter particles become quantum blackholes giving away photons that match the typical photons in the radiation background. It does not seem to make sense to go below r_p/c, given that this is Planck's time at this age.

7. DISTANCE VERSUS RED SHIFT RELATION

The luminosity distance d_L of a light source is defined as

$$d_L = \frac{L}{4\pi l}^{1/2} \tag{80}$$

and if we know the absolute luminosity L of the sources and measure their apparent luminosity l we can find d_L. By measuring the red shifts z an empirical curve for $d_L(z)$ is obtained. The theoretical expression is derived using a power series for the cosmic scale factor

$$R(t_1) = R(t_0)[1 + H_0(t_1 - t_0) - \tfrac{1}{2}q_0H_0^2(t_1 - t_0)^2 + \cdots] \tag{81}$$

where t_0 is the present moment and t_1 the time at emission from a light source at coordinate distance r_1, and H_0 and q_0 are the Hubble constant and the deacceleration parameter, respectively, given by

$$H_0 = \frac{\dot{R}(t_0)}{R(t_0)} \tag{82}$$

$$q_0 = -\ddot{R}(t_0)\frac{R(t_0)}{\dot{R}^2(t_0)} \tag{83}$$

The observables d_L and z are related to t_1 and r_1 by the general relativity theoretical relations

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = \int_0^{r_1} \frac{1}{(1-kr^2)^{1/2}}\, dr \tag{84}$$

$$z = \frac{\lambda_0}{\lambda_1} - 1 = \frac{\nu_1}{\nu_0} - 1 = \frac{R(t_0)}{R(t_1)} - 1 \tag{85}$$

$$d_L = r_1 \frac{R^2(t_0)}{R(t_1)} \tag{86}$$

where the classical cosmic red shift law $\nu R = \text{const}$ has been used. By expanding in power series in terms of $t_0 - t_1$ one arrives at the well-known luminosity distance versus red shift relation, in power series of z

$$d_L = \frac{1}{H_0}\left[z + \frac{1}{2}(1-q_0)z^2 + \cdots \right] \tag{87}$$

Estimates of the observed data imply a value of $H_0 = 50\,\text{km sec}^{-1}\,\text{Mpc}^{-1}$ and $q_0 = 1.0$ that have been used by many authors (e.g., Land, Lord, Johanson, and Savage, 1975). Here we will find the equivalent to (87) with creation present.

With G varying with time we have creation and the resultant cosmic red shift law is (68)

$$\frac{\nu R}{N_\gamma^{1/3}} = \text{const} \tag{68}$$

Let us call C_r the creation factor affecting red shift, defined as

$$C_r = \left(\frac{N\gamma_0}{N\gamma} \right)^{1/3} \tag{88}$$

and (68) is then

$$C_r \nu R(t) = \nu_0 R(t_0) \tag{89}$$

Let us find the luminosity distance versus red shift relation taking into account creation. The relevant relations are now

$$z = \frac{\lambda_0}{\lambda_1} - 1 = \frac{\nu_1}{\nu_0} - 1 = \frac{R(t_0)}{R(t_1)} \frac{1}{C_r} - 1 \tag{90}$$

instead of (85). Since creation implies higher energy photons arriving at the observer by the factor C_r in (89), the apparent luminosity l will be increased by C_r in (80). Taking this into account the relevant relation in place of (86) is now

$$d_L = r_1 \frac{R^2(t_0)}{R(t_1)} \frac{1}{C_r^{1/2}} \tag{91}$$

From (90) and (91) we get

$$d_L = r_1 R(t_0)(1+z)C_r^{1/2} \tag{92}$$

Let us assume a power law for C_r

$$C_r = \left(\frac{t_0}{t_1}\right)^\alpha = 1 + \alpha \frac{t_0 - t_1}{t_0} + \alpha \frac{\alpha+1}{2}\left(\frac{t_0 - t_1}{t_0}\right)^2 + \cdots \tag{93}$$

where in our case $G \div t^{-1}$ giving $N \div t^{3/4}$ from (67) implies

$$\alpha = \tfrac{1}{4} \tag{94}$$

Let us call H_0^c and q_0^c the Hubble constant and the deacceleration parameter in the case of a Universe with creation. By using (81) we expand (84) to obtain

$$r_1 = \frac{1}{R(t_0)}\left[t_0 - t_1 + \frac{1}{2} H_0^c (t_0 - t_1)^2 + \cdots\right] \tag{95}$$

while (81), (90), and (93) give a power series for the red shift as a function of the time of flight $t_0 - t_1$

$$\begin{aligned} z = \left(H_0^c - \frac{\alpha}{t_0}\right)(t_0 - t_1) + \Bigg[(H_0^c)^2\left(\frac{1}{2} q_0^c + 1\right) - \alpha \frac{(\alpha+1)}{2t_0^2} \\ - \left(H_0^c - \frac{\alpha}{t_0}\right)\frac{\alpha}{t_0}\Bigg](t_0 - t_1)^2 + \cdots \end{aligned} \tag{96}$$

To obtain the time of flight in terms of the red-shift we invert the above power series

$$\begin{aligned} t_0 - t_1 &= \frac{t_0}{H_0^c t_0 - \alpha} z - \left(\frac{t_0}{H_c^c t_0 - \alpha}\right)^3 \Bigg[(H_0^c)^2\left(\frac{1}{2} q_0^c + 1\right) - \frac{\alpha(\alpha+1)}{2t_0^2} \\ &\quad - \left(H_0^c - \frac{\alpha}{t_0}\right)\frac{\alpha}{t_0}\Bigg] z^2 + \cdots \\ &= az + bz^2 + \cdots \end{aligned} \tag{97}$$

so that (95) gives in terms of z

$$r_1 = \frac{1}{R(t_0)}\left[az + \left(b + \frac{1}{2} H_0^c a^2\right) z^2 + \cdots\right] \tag{98}$$

and (93) for $C_r^{1/2}$

$$C_r^{1/2} = 1 + \frac{\alpha_a}{2t_0} z + \left[\frac{\alpha_b}{2t_0} + \frac{\alpha(2+\alpha)}{8t_0^2} a^2\right] z^2 + \cdots \tag{99}$$

Hence, the luminosity distance (92) has the power series expansion in terms of z, using (98) and (99)

$$d_L = az + \left(a + b + \frac{1}{2} H_0^c a^2 + \frac{\alpha a^2}{2t_0}\right) z^2 + \cdots \tag{100}$$

and substituting the coefficients a and b from (97) we finally arrive at

$$d_L = \frac{t_0}{H_0^c t_0 - \alpha}\left(z + \left\{1 + \frac{t_0}{H_0^c t_0 - \alpha}\left(\frac{1}{2} H_0^c + \frac{3}{2}\frac{\alpha}{t_0}\right) - \left(\frac{t_0}{H_0^c t_0 - \alpha}\right)^2 \left[(H_0^c)^2\left(\frac{1}{2} q_0^c + 1\right) - \frac{\alpha(\alpha+1)}{2t_0^2}\right]\right\} z^2\right) \tag{101}$$

Hence, the experimental observation that the coefficient of z has the value H_{obs}^{-1} gives in the case of creation

$$H_{\text{obs}} = H_0^c - \frac{\alpha}{t_0} \tag{102}$$

and the experimental observation that the coefficient of z^2 has the value $H_{\text{obs}}^{-1}\frac{1}{2}(1 - q_{\text{obs}})$, from (87), gives, using (102),

$$q_{\text{obs}} H_{\text{obs}}^2 + \frac{\alpha(1-\alpha)}{t_0^2} = \left(H_{\text{obs}} + \frac{\alpha}{t_0}\right)^2 q_0^c \tag{103}$$

where in (102) and (103) with no creation ($\alpha = 0$) one has $H_{\text{obs}} = H_0^c$ and $q_{\text{obs}} = q_0^c$ as it should. It is seen that the experimental results provide two relations, (102) and (103), that determine the Hubble constant and the deacceleration parameter in the case of creation if t_0 is known.

8. COSMOLOGICAL MODEL

Let us take a Friedmann model. The fundamental equations of dynamical cosmology are the Einstein equations (e.g., Weinberg, 1972)

$$3\ddot{R} = -4\pi G(\rho + 3p)R \tag{104}$$

$$R\ddot{R} + 2\dot{R}^2 + 2K = 4\pi G(\rho - p)R^2 \tag{105}$$

the energy equation (7)

$$Gpd(R)^3 + d(G\rho R^3) = 0 \tag{7}$$

and the equation of state. By eliminating $\ddot{R}$ from (104) and (105) one finds a first-order differential equation for $R(t)$

$$\dot{R}^2+K=\tfrac{8}{3}\pi G\rho R^2 \tag{106}$$

with $K=0$, -1 or $+1$. One can show that if we find a solution for $R(t)$ satisfying (106) and (7), then (104) and (105) are automatically satisfied.

Let us analyze the energy equation (7). If the photon distribution function has been propagating without collisions for most of the age of the Universe, then the cosmic red shift law (57) gives, with typical photons of energy $h\nu$, the relation for radiation density ρ_γ

$$G\rho_\gamma R^4=\text{const} \tag{107}$$

Using the free matter particle propagation result (39), which for constant m_p gives $GN_pm_p=\text{const}$, one has in terms of density of matter particles ρ_p

$$G\rho_p R^3=\text{const} \tag{108}$$

Idealizing the Universe as containing free matter particles and photons one has for the pressure and the density

$$p=p_p+p_\gamma=p_p+\tfrac{1}{3}\rho_\gamma \tag{109}$$

$$\rho=\rho_p+\rho_\gamma \tag{110}$$

and substituting into the energy equation (7) taking into account (107) and (108) one has

$$p_p d(R^3)=0, \qquad \text{i.e., } p_p=0 \tag{111}$$

The above result tells us that the idealization made of a Universe consisting of free matter particles and photons implies that the matter particle pressure is zero, a condition satisfied at present and generally used when considering $p_p \ll \rho_p$, i.e., that matter particles are nonrelativistic. We assume that this has been always true, which is equivalent to saying that matter particles are created with very small linear momentum throughout most of the age of the Universe. The assumption may not be valid at an early stage of the Universe where the de Broglie wavelength of matter particles is smaller. However, at an early stage of the Universe radiation dominates, and matter particle pressure and density do not affect the field equation for R, so that the assumption of zero pressure for matter particles is not necessary. To substantiate this consideration when creation is present, let us find the ratio ρ_γ/ρ_p. From (107) and (108)

$$\frac{\rho_\gamma}{\rho_p}=\frac{R_0}{R}\frac{\rho_\gamma^0}{\rho_p^0} \tag{112}$$

where the index 0 indicates conditions at present $t=t_0$. Equation (112) implies that the ratio of photon to matter energy is not affected by creation, except through R. If we go back in time, the Universe is radiation dominated for $R<R_e$, where R_e is given by the condition $\rho_\gamma=\rho_p$ in (112):

$$R_e=\frac{\rho_\gamma^0}{\rho_0^0}R_0 \tag{113}$$

With (107) and (108) we can write the differential equation (106) for $R(t)$ as

$$\dot{R}^2+K=\frac{A}{R}+\frac{B}{R^2} \tag{114}$$

where A and B are two universal constants given by

$$A=\tfrac{8}{3}\pi G\rho_p R^3=\tfrac{8}{3}\pi G_0\rho_p^0 R_0^3 \tag{115}$$

$$B=\tfrac{8}{3}\pi G\rho_\gamma R^4=\tfrac{8}{3}\pi G_0\rho_\gamma^0 R_0^4 \tag{116}$$

Before solving (114) let us analyze the present state of observations to determine K through the condition of zero matter pressure in the case of creation. With superindex c standing for creation, Hubble's constant H_0^c and the deacceleration parameter q_0^c defined as

$$H_0^c=\frac{\dot{R}_0}{R_0} \tag{117}$$

$$q_0^c=-\frac{\ddot{R}_0R_0}{\dot{R}_0^2} \tag{118}$$

can be used to obtain the present energy density and pressure. From (104) and (105), taking into account that at present the Universe is matter dominated, one has

$$\rho_p^0=\frac{3}{8\pi G}\left[\frac{K}{R_0^2}+(H_0^c)^2\right] \tag{119}$$

$$p_p^0=-\frac{1}{8\pi G}\left[\frac{K}{R_0^2}+(H_0^c)^2(1-2q_0^c)\right] \tag{120}$$

The zero pressure condition for matter particles implies from (120)

$$\frac{K}{R_0^2}=(H_0^c)^2(2q_0^c-1) \tag{121}$$

Using the results found from red shifts and luminosities in the previous section, (102) and (103) in (121), one obtains

$$\frac{K}{R_0^2}+\left(H_{\text{obs}}+\frac{\alpha}{t_0}\right)^2=2\left[q_{\text{obs}}H_{\text{obs}}^2+\frac{\alpha(1-\alpha)}{t_0^2}\right] \tag{122}$$

or equivalently

$$\left(\frac{1}{t_0}\right)^2\alpha(2-3\alpha)-\frac{1}{t_0}2H_{\text{obs}}\alpha+H_{\text{obs}}^2(2q_{\text{obs}}-1)-\frac{K}{R_0^2}=0 \tag{123}$$

which is the condition for zero pressure with creation. The condition that this equation must have real roots for $1/t_0$ implies

$$K\geq 2H_{\text{obs}}^2R_0^2\left(q_{\text{obs}}-\frac{1-\alpha}{2-3\alpha}\right) \tag{124}$$

and for $\alpha=1/4$ as in (94) one gets

$$K\geq 2H_{\text{obs}}^2R_0^2\left(q_{\text{obs}}-\frac{3}{5}\right) \tag{125}$$

so that $K=0$ for $q_{\text{obs}}=0.6$, $K=-1$ for $q_{\text{obs}}<0.6$ and $K=1$ for $q_{\text{obs}}>0.6$. Adopting the value $q_{\text{obs}}=1.0$, then $K=1$ and we are dealing with a finite, unbounded Universe, with proper volume

$$V=2\pi^2R^3(t) \tag{126}$$

which is the well-known Einstein solution to cosmology.

Having obtained $K=1$ we now determine and solve the equations to be satisfied by the present conditions t_0, R_0, ρ_p^0, and $\dot{R}_0$. The condition for zero pressure with creation (123) with $q_{\text{obs}}=1.0$ and $K=1$ is then

$$\left(\frac{1}{t_0}\right)^2\alpha(2-3\alpha)-\frac{1}{t_0}2H_{\text{obs}}\alpha+H_{\text{obs}}^2-\frac{1}{R_0^2}=0 \tag{127}$$

For present conditions, $t=t_0$, we also have equation (102) from the red shift observation,

$$H_{\text{obs}}=\frac{\dot{R}_0}{R_0}-\frac{\alpha}{t_0} \tag{128}$$

together with (114), with $K=1$, that gives

$$\dot{R}_0^2+1=\frac{A}{R_0}+\frac{B}{R_0^2}\simeq\frac{A}{R_0} \tag{129}$$

where we have used $\rho_\gamma^0\ll\rho_p^0$, which implies $B\ll AR_0$. Equations (127), (128), and (129) must be satisfied for present conditions of t_0, R_0, ρ_p^0, and $\dot{R}_0$, with $H_{\text{obs}}=50\text{ km sec}^{-1}\text{ Mpc}^{-1}\simeq(2\times10^{10}\text{ yr})^{-1}$. In order to get solutions independent of the particular value of H_{obs}, let us call

$$x = H_{obs} \cdot R_0$$
$$y = H_{obs} \cdot t_0 \tag{130}$$
$$z = \frac{8}{3}\pi G\rho_p^0 H_{obs}^{-2} = \frac{AH_{obs}}{x^3}$$

and (127), (128), and (129) are now in terms of (130)

$$\frac{1}{y^2}(2-3\alpha)\alpha - \frac{1}{y}2\alpha + 1 - \frac{1}{x^2} = 0$$
$$\frac{\dot{R}_0}{x} - \frac{\alpha}{y} = 0 \tag{131}$$
$$\dot{R}_0^2 + 1 = zx^2$$

There is one more relation that t_0 and R_0 must satisfy. Integrating the field equation (114) with $K=1$ and with the condition $R=0$ for $t=0$, we get the solution

$$t = B^{1/2} - (-R^2 + AR + B)^{1/2} + \frac{A}{2}\left[\sin^{-1}\frac{2R-A}{(A^2+4B)^{1/2}} + \sin^{-1}\frac{A}{(A^2+4B)^{1/2}}\right] \tag{132}$$

Taking into account that $AR_0 \gg B$ and $A^2 \gg B$ (132) gives in terms of x, y, and z defined in (130),

$$y = -x(zx^2-1)^{1/2} + \frac{zx^3}{2}\sin^{-1}\frac{2(zx^2-1)^{1/2}}{zx^2} \tag{133}$$

which is the relation that together with (131) define x, y, z, and $\dot{R}_0$. Table I presents the relevant solutions for different values of the parameter α. Table II presents the corresponding values of R_0, t_0, and ρ_p^0 in terms of α

Table I

α	x	y	z	$\dot{R}_0$	AH_{obs}
0.10	0.868	0.4775	2.789	1.050	1.824
0.15	0.802	0.4314	3.370	1.081	1.738
0.20	0.736	0.3857	4.151	1.118	1.655
0.25	0.670	0.3404	5.236	1.166	1.575
0.30	0.604	0.2957	6.803	1.216	1.499
0.35	0.537	0.2516	9.188	1.284	1.423

Table II

α	R_0 (10^9 yr)	t_0 (10^9 yr)	ρ_p^0(10^{-29} g cm^{-3})
0.10	17.4	9.55	1.25
0.15	16.0	8.63	1.51
0.20	14.7	7.71	1.87
0.25	13.4	6.81	2.35
0.30	12.1	5.91	3.06
0.35	10.7	5.03	4.13

for $H_{\text{obs}}^{-1} = 2\times 10^{10}$ yr. From Table II and (130) the corresponding values for a law $G \div t^{-1}$ ($\alpha = 0.25$) are

$$\begin{aligned} R_0 &= 1.34\times 10^{10}\ \text{yr} \\ t_0 &= 6.81\times 10^{9}\ \text{yr} \\ \rho_p^0 &= 2.35\times 10^{-29}\ \text{g/cm}^3 \end{aligned} \tag{134}$$

For the present number of particles in the Universe we get, using (126)

$$N_p^0 = \frac{\rho_p^0 V_0}{m_p} = \frac{9.47\times 10^{56}\ \text{g}}{m_p} = 5.67\times 10^{80}\ \text{particles} \tag{135}$$

while for the number of photons due to the 2.7-K background

$$N_\gamma^0 = 20 T_0^3 V_0 = 1.59\times 10^{88}\ \text{photons} \tag{136}$$

Let us find the conditions of the Universe at an early age. For very small values of t or, what is the same, of R, the field equation (114) becomes independent of K and A, i.e.,

$$\dot{R}^2 \simeq \frac{B}{R^2} \tag{137}$$

and integrating one gets

$$R^2 = 2(B)^{1/2} t = 2(\tfrac{8}{3}\pi G_0 \rho_\gamma^0 R_0^4)^{1/2} t \tag{138}$$

where (116) has been used. Substituting present values for G_0 and $\rho_\gamma^0 = 4.40\times 10^{-34}$ g/cm^3 we get, with t in seconds,

$$\frac{R}{R_0} = 1.77\times 10^{-10}\ t^{1/2} \tag{139}$$

At $t_i = 10^{-40}\ t_0 = 2.15\times 10^{-23}$ sec one has for R_i

$$\frac{R_i}{R_0} = 8.2\times 10^{-22} \tag{140}$$

Now, the present radiation background has a typical wavelength $\lambda_0 =$ 1.4 mm. The red shift law (70) expressed in terms of wavelength is

$$\lambda = \lambda_0 \frac{R}{R_0}\left(\frac{t_0}{t}\right)^{1/4} \tag{141}$$

which means that at t_i, using (140) this radiation had a wavelength

$$\lambda_i = 1.1 \times 10^{-12} \text{ cm} \tag{142}$$

This result is to be compared with (76): In the previous section we have shown that, owing to the time dependence of the Planck units through G, at $t_i = 10^{-40} t_0$ the matter particles are quantum black holes and therefore evaporate, giving blackbody radiation of typical wavelength (76), 1.8×10^{-12} cm. We now see that this radiation red shifted to the present approximately gives the 2.7-K background radiation. The picture at that age is a sea of quantum blackholes, photons, and matter particles, immersed in a volume given by (134) and (140) as

$$V_i = 2\pi^2 R_i^3 = 2.2 \times 10^{22} \text{ cm}^3 \tag{143}$$

The temperature corresponding to λ_i in (142) is

$$T_i \simeq 3 \times 10^{11} \text{ K} \tag{144}$$

The value of R_e at which $\rho_p = \rho_\gamma$ is obtained taking into account (113) and (134)

$$R_e = 2.51 \times 10^5 \text{ yr} \tag{145}$$

and (132) with $A = 3.15 \times 10^{10}$ yr and $B = 7.90 \times 10^{15}$ yr^2 gives for t_e

$$t_e = 5.2 \times 10^7 \text{ yr} \tag{146}$$

Finally, the solution to the field equation for $R(t)$ given by (132) gives the well-known expansion and contraction model with maximum $R = A$ at $t = (\pi/2)A$ and $R = 0$ again at $t = \pi A$. The values of A in Table I in terms of the creation exponent α tell us that the more important creation is the smaller is the time scale for the Universe, i.e., evolution occurs faster.

9. DISCUSSION

The application of the field-to-particle technique for free matter particles proves that creation does not affect their propagation and that one must have $GNm_p = \text{const}$ for any proper volume. With the model of the Universe $K = 1$, which is consistent with the luminosity red shift experimental results, one has a finite volume and mass in the Universe. One can interpret G as being inversely proportional to the rest mass (energy) of the

Universe. Hence, Einstein's field equations contain, through G, the cosmological parameter $N_p m_p$ and therefore the presence of all matter in the Universe is explicit in the field equations.

The application of the field-to-particle technique for free photons proves that creation affects their propagation and that one must have $GN_\gamma \nu R = \text{const}$, which is the expression for the cosmic red shift law with creation. Conservation of the background photon distribution function imposes that the photon creation rate must be such that $G^3 N_\gamma^4 = \text{const}$. Hence one has the cosmic red shift law with creation as $\nu R N_\gamma^{-1/3} = \text{const}$ or $G^{1/4} \nu R = \text{const}$ in terms of N_γ or G, respectively.

Idealizing the Universe as formed by free matter particles and free photons, one can use the results of the field-to-particle technique into the energy equation that is then satisfied for zero matter particle pressure, a condition that is observed to be very approximately true at present. Assuming that this has always been the case one can readily integrate the field equation defining the cosmic scale factor $R(t)$.

Once the distance versus red shift relation is derived including creation, the use of the luminosity-red-shift observations, $H_{\text{obs}} = 50$ km sec^{-1} Mpc^{-1} and $q_{\text{obs}} = 1.0$, completely determines the model of the Universe in terms of the creation exponent α. The application for the case $\alpha = 0.25$, i.e., $G \div t^{-1}$, gives a present age for the Universe $t_0 = 6.81 \times 10^9$ yr. In a review of the cosmic time scale Peebles (1971) discussed three age measurements: Hubble's constant, stellar evolution, and radioactive decay of the elements. With creation $H_0^c = \dot{R}_0 / R_0$ is greater than H_{obs} according to equation (128), so that based on H_{obs} we have found the age of the Universe as $t_0 = 6.81 \times 10^9$ yr instead of $(\pi/2 - 1) H_{\text{obs}}^{-1} = 1.1 \times 10^{10}$ yr, which is the value of t_0 with no creation, $\alpha = 0$. Our result agrees with the uranium model age, which gives a value of 6.65×10^9 yr for prompt production of uranium. On the other hand stellar evolution ages are greater than these values but, with creation, G is decreasing with time and the stellar evolution rate must have been greater in the past. This means that the creation model presented here gives an age for the Universe that brings together the three age measurements known. With less creation, i.e., α decreasing, one obtains a higher present age.

The model gives a density $\rho_p^0 = 2.35 \times 10^{-29}$ g/cm^3 which implies the well-known missing-mass problem when compared with the mean mass density due to galaxies, from mass-luminosity results, which is about 10^{-31} g/cm^3 for $H_{\text{obs}}^{-1} = 2 \times 10^{10}$ yr. This suggests, in a creation model, that creation is taking place in intergalactic space, most of the mass of the Universe being there, and with particles having a very small momentum.

For the case $G \div t^{-1}$ the role of Planck's units becomes very important at age $t_i = 10^{-40} t_0$, which is also Planck's time at t_i. The coincidence of

Planck's units at t_i with the proton's characteristics strongly suggests an origin of the Universe out of quantum blackholes at t_i. Their possible evaporation output in terms of blackbody radiation that also coincides with the present 2.7-K radiation background, after red shifting from t_i to t_0, is another argument in support of this view. The picture is one of continuous creation of matter particles and photons of an intensity defined by $GN_p =$ const and $GN_\gamma^{4/3} =$ const.

The two universal constants A and B defined in (115) and (116) give the evolution of the Universe, A setting the overall time scale and B the behavior close to the beginning and the end of the expansion-contraction cycle.

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